

Fermion-induced quantum critical points in 2D Dirac semimetals

Shao-Kai Jian and Hong Yao

Institute for Advanced Study, Tsinghua University, Beijing 100084, China

(Dated: October 26, 2016)

In this paper we investigate the nature of quantum phase transitions between 2D Dirac semimetals and Z_3 -ordered phases (e.g. Kekule valence-bond solid), where cubic terms of the order parameter are allowed in the quantum Landau-Ginzberg theory and the transitions are putatively first-order. From large- N RG analysis, we find that fermion-induced quantum critical points (FIQCPs), which are type-II Landau-forbidden transitions introduced in arXiv:1512.07908, occur when N (the number of flavors of 4-component Dirac fermions) is larger than a critical value N_c . Remarkably, from the knowledge of spacetime supersymmetry, we obtain an *exact* lower bound for N_c , *i.e.*, $N_c > 1/2$. (Here “1/2” flavor of 4-component Dirac fermions is equivalent to one flavor of 4-component Majorana fermions). Moreover, we show that the emergence of two length scales is a typical phenomenon of FIQCP and obtain two different critical exponents, e.g. $\nu \neq \nu'$, by large- N RG calculations. We further give a brief discussion on possible experimental realizations of FIQCP.

I. INTRODUCTION

Examples of type-II Landau-forbidden phase transitions, which violate the Landau cubic-term criterion saying that a phase transition must be first-order if cubic terms are allowed in the Landau-Ginzberg free energy [1–4], include exactly solvable three-state Potts model in 1+1D [5, 6]. No higher dimensional type-II Landau-forbidden transitions were identified until fermion-induced quantum critical points (FIQCPs) were introduced recently in Ref. [7], where a putative first-order phase transition can be driven to be continuous by gapless Dirac fermions in 2+1D. In Ref. [7], a $SU(N)$ fermionic model featuring a Z_3 transition from 2D Dirac semimetals to Kekule valence bond solids (Kekule-VBS), where cubic terms of the order parameters are allowed by symmetry, was conceived and studied using a large-scale sign-problem-free Majorana quantum Monte Carlo simulations (QMC) [8, 9] and large- N renormalization group (RG) calculations [10–14]. Both QMC simulations and RG analysis show the occurrence of a FIQCP when N , the number of flavors of 4-component Dirac fermions, is larger than a critical value N_c . This FIQCP scenario was also confirmed later by $4-\epsilon$ RG calculations [15].

It has been known that the number of flavors of Dirac fermions is often of crucial importance in determining the nature of low-energy physics and quantum phase transitions [16]. For example, the stability of algebraic spin liquids in 2+1D [17–26] [equivalently, the stability of a deconfined phase of the compact quantum electrodynamics (QED) in 2+1D] depends essentially on the number of flavors of Dirac fermions [27–32]; another related issue is the critical number of flavors for chiral symmetry breaking in non-compact QED [32–42]. For the quantum phase transitions between the 2D Dirac semimetals and a broken Z_3 symmetry phase with massive Dirac fermions, which are called Z_3 Gross-Neveu transitions, the large- N one-loop RG calculations [7] give rise to $N_c = \frac{1}{2}$ such that only when $N > N_c$ can a FIQCP occur. A schematic quantum phase diagram for the occurring of such a FIQCP is shown in Fig. 1. Large- N RG calculations show that

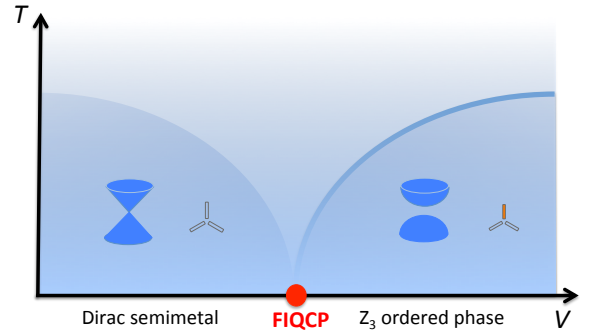


FIG. 1. A schematic phase diagram for a phase transition from a gapless Dirac semimetal to a Z_3 ordered phase that is presented by a Kekule valence bond solids. A fermion-induced quantum critical points (FIQCP) is presented in the zero-temperature.

FIQCP cannot occur at Z_3 Gross-Neveu transitions in 3+1D for any finite N .

Nonetheless, it is desired to obtain non-perturbative and even exact results about N_c , the critical number of flavors of Dirac fermions for the occurrence of FIQCPs at Z_3 Gross-Neveu transitions in 2+1D. In this paper, with the help of spacetime supersymmetry (SUSY) [44–48], we can show *exactly* that for $N = \frac{1}{2}$ such Z_3 Gross-Neveu transition must be first-order. Consequently, we obtain an exact lower bound for N_c , namely $N_c > \frac{1}{2}$.

Another interesting aspect of FIQCP is the emergence of two length scales [49–55] due to the presence of dangerously irrelevant operators at the transition point, which is a typical phenomenon of Landau-forbidden phase transitions. For instance, two length scales are found at deconfined quantum critical points (DQCP) [55–61], which are type-I Landau-forbidden phase transition. FIQCPs, as a scenario of type-II Landau-forbidden transitions, also exhibit the emergence of two length scales owing to the dangerously irrelevant cubic terms at the critical points. A consequence of emergent two length scales is that critical exponents can be different on the two sides of the transition. We obtain these different critical exponents

by one-loop RG calculations.

The paper is organized as follows. In Sec. II, the low-energy effective theory describing the Z_3 phase transitions is constructed by symmetry and scaling considerations. In Sec. III, a large- N RG analysis is carried out for the Z_3 Gross-Neveu transitions in $d+1$ dimensions, where N is the number of four-component Dirac fermions and $2 \leq d \leq 3$. Critical spatial dimensions for occurrence of Z_3 Gross-Neveu transitions are discussed. One-loop RG analysis shows that FIQCPs can occur for $N > 1/2$ when $d = 2$ but cannot for any N when $d = 3$. In Sec. IV, we construct field theory with a single Dirac cone (two Majorana cones), i.e., $N = \frac{1}{2}$, interacting with $U(1)$ order parameter in 2+1D [62], where the FIQCP is assumed to occur by ignoring the cubic anisotropy first. The fixed point theory has emergent SUSY in 2+1D [63–68], corresponding to the SUSY Wess-Zumino model after a particle-hole transformation. However, cubic perturbations at such SUSY fixed point turn out to be relevant, rendering the phase transition discontinuous and ruining the assumed FIQCP. Consequently, this provides an exact lower bound $N_c > \frac{1}{2}$ for occurrence of FIQCP at Z_3 Gross-Neveu transitions in 2+1D. In Sec. V, we investigate emergent two length scales at FIQCP. We obtain different critical exponents on both sides of transition. In the end, we conclude these theoretical investigations and give a brief discussion on possible experimental realizations of FIQCPs in Sec. VI.

II. THE EFFECTIVE THEORY FOR Z_3 GROSS-NEVEU TRANSITIONS IN DIRAC SEMIMETALS

The non-interacting Dirac fermion in d dimensions (d is the spatial dimension) is well known and given by

$$\mathcal{L}_\psi = \psi^\dagger (\partial_\tau - iv\gamma^\mu \partial_\mu) \psi, \quad (1)$$

where summation over $\mu = 1, \dots, d$ is assumed implicitly, the Gamma matrices are $2^{\lfloor \frac{d}{2} \rfloor + 1}$ dimensional where $[x]$ denotes the integer part of x and satisfy $\gamma^{\mu\dagger} = \gamma^\mu$, $\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}$. Here v is the Fermi velocity and τ is imaginary time. The Dirac fermions presented here have rotational symmetry, i.e., the Fermi velocity is assumed to be isotropic, which is an emergent symmetry in the infrared [69–71]. A concrete 4×4 matrix representation of Gamma matrices in $d = 2$ can be given by $\gamma^1 = \sigma^x \tau^z$, $\gamma^2 = \sigma^y$, where σ and τ are Pauli matrices. For spinless fermions in 2D honeycomb lattice with nearest neighbor (NN) hopping, the Pauli matrices σ (τ) represent sub-lattice (valley) indices.

To gap out Dirac fermions by the Z_3 order, $d+2$ anti-commuting Gamma matrices are necessary: d Gamma matrices represent the relevant degrees of freedom of d -dimensional Dirac fermions shown above, while the other two Gamma matrices, γ^{d+1} and γ^{d+2} , fulfill two mass terms related to the Z_3 order parameters [59, 72]. In this representation, the Z_3 symmetry is generated by $T =$

$\exp[\frac{\pi}{3}\gamma^{d+1}\gamma^{d+2}]$. Thus the lowest order coupling term is

$$\mathcal{L}_{\psi\phi} = g(\phi\psi^\dagger\gamma^+\psi + \phi^*\psi^\dagger\gamma^-\psi), \quad (2)$$

where ϕ is a complex order parameter, and $\gamma^\pm = \frac{1}{2}(\gamma^{d+1} \pm i\gamma^{d+2})$, and g is a real constant describing the strength of fermion-boson coupling. The fermion-boson coupling has $O(2)$ symmetry larger than Z_3 . For the transition between the Dirac semimetal to Kekule-VBS on the honeycomb lattice, two Gamma matrices are $\gamma^3 = \sigma^x \tau^x$, $\gamma^4 = \sigma^x \tau^y$. The order parameter is given by the Kekule valence bonds density with $2\vec{K}$ momentum, where \vec{K} is the valley momentum and the Z_3 symmetry corresponds to the translation symmetry in graphene.

For the LG theory of the order parameter, the Lagrangian dictated by symmetries reads

$$\mathcal{L}_\phi = |\partial_\tau \phi|^2 + c^2 |\nabla \phi|^2 + r|\phi|^2 + b(\phi^3 + \phi^{*3}) + u|\phi|^4, \quad (3)$$

where r is the effective tuning parameter for phase transition, b is real constant representing the strength of cubic terms and u is assumed to be positive. Again, the isotropy of boson velocity c is a low-energy emergent phenomenon. Note that the cubic term is allowed by symmetries and lowers the $O(2)$ symmetry of LG theory to Z_3 and putatively renders a first-order phase transition according to the Landau cubic-term criterion. An effective particle-hole symmetry ($\phi \rightarrow \phi^*$, e.g., reflection symmetry in the Kekule transition) is also assumed to exclude the first-order time derivative of the order parameters.

III. LARGE- N RENORMALIZATION GROUP ANALYSIS FOR Z_3 GROSS-NEVEU TRANSITIONS IN DIRAC SEMIMETALS

A. RG equations in $d + 1$ dimensions

The ultraviolet degrees of freedom (fast modes) constrained in the momentum shell $\Lambda e^{-l} < p < \Lambda$, where $l > 0$ is flow parameter and Λ is a cutoff, are integrated out to generate the RG equations. In the following calculations, we promote the Dirac fermions to have N flavors and use 4×4 Gamma matrices for simplicity since we are only interested in $d = 2, 3$. Various trace properties for Gamma matrices are given in the Appendix A. Non-vanishing one-loop Feynman diagrams are shown in Fig. 2. The critical r_c is renormalized to be $-\frac{8}{N}$ and won't affect the fixed point properties at the lowest level (see Appendix B for details). So we set $r = 0$ in following calculations for simplicity.

The renormalizations of boson and fermion self-energy come from Fig. 2(a), 2(b) and 2(c). After evaluating the Feynman diagrams 2(a), 2(b), the boson self-energy reads

$$\Pi(p) = g^2 \frac{N\pi}{2v^3} K_d(\omega^2 + v^2 p^2) + b^2 \frac{9\pi}{4c^5} K_d \Lambda^{d-5} l(\omega^2 - \frac{1}{3} c^2 p^2), \quad (4)$$

where $K_d \equiv \frac{A_d}{(2\pi)^{d+1}}$, A_d is the area of unit d -sphere and $p^2 \equiv \sum_{\mu=1}^d p_\mu^2$. Feynman diagram 2(c) gives rise to fermion self-energy,

$$\Sigma(p) = \frac{g^2 \pi K_d \Lambda^{d-4} l}{c(c+v)^2} (-i\omega + \frac{2c+v}{3v} \gamma^\mu p_\mu). \quad (5)$$

From the self-energy parts, the RG equations for boson and fermion velocities are given by

$$\frac{dc}{dl} = -\tilde{g}^2 \frac{N\pi(c+v)}{4c^3 v^3} (c-v) - \tilde{b}^2 \frac{3\pi}{2c^6}, \quad (6)$$

$$\frac{dv}{dl} = -\tilde{g}^2 \frac{2\pi(v-c)}{3cv(c+v)^2}, \quad (7)$$

where dimensionless coupling constants are defined as $\tilde{g}^2 \equiv K_d \Lambda^{d-3} g^2$, $\tilde{b}^2 \equiv K_d \Lambda^{d-5} b^2$, $\tilde{u} \equiv K_d \Lambda^{d-3} u$. The anomalous dimensions for both boson and fermion fields are $\eta_\phi = \frac{N\pi\tilde{g}^2}{4v^3} + \frac{9\pi\tilde{b}^2}{8c^5}$, $\eta_\psi = \frac{\pi\tilde{g}^2}{2c(c+v)^2}$. For a continuous phase transition, one demands $\tilde{b}^2 = 0$ at fixed point. In such case, the velocities of fermions and bosons will flow to the same, as seen in the following RG equations,

$$\frac{d(c-v)}{dl} = -\tilde{g}^2 \left[\frac{N\pi(c+v)}{4c^3 v^3} + \frac{2\pi}{3cv(c+v)^2} \right] (c-v). \quad (8)$$

The rest of Feynman diagrams generate the RG equations for coupling constants. From Feynman diagram Fig. 2(d), the three-boson vertex is

$$\Gamma_{\phi^3} = \Gamma_{\phi^*3} = -bu \frac{3\pi}{c^3} K_d \Lambda^{d-3} l. \quad (9)$$

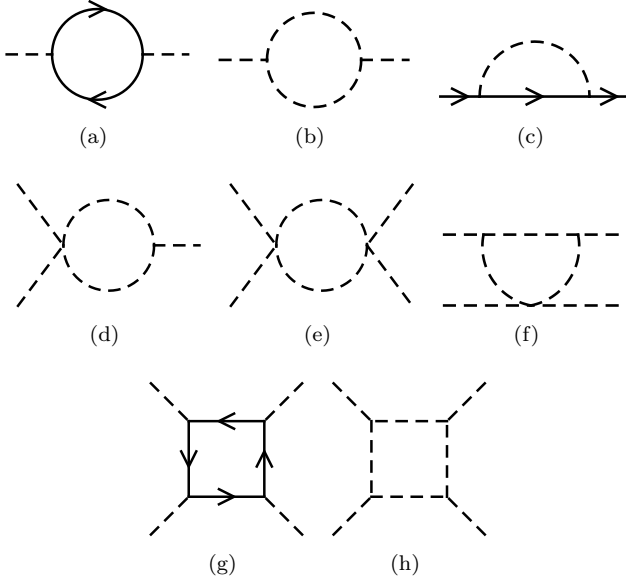


FIG. 2. One-loop Feynman diagrams. The arrowed solid line indicates fermion propagator and dashed line indicates boson propagator.

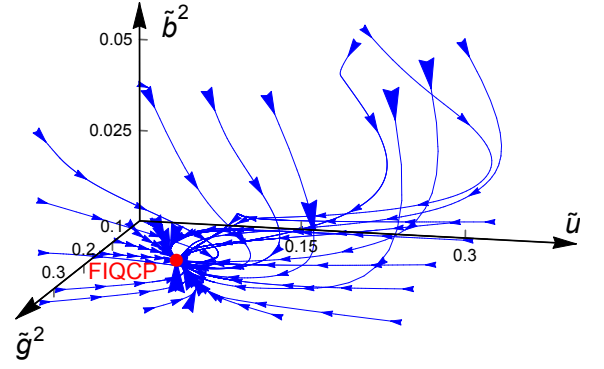


FIG. 3. The flow diagram in three dimensions generated by the RG equations for $N=3$. The blue arrows indicate the flow of coupling constants when energy is lowered. The red point located in the \tilde{g}^2 - \tilde{u} plane is a stable fixed point corresponding to FIQCP.

By evaluating Fig. 2(e), 2(f), 2(g) and 2(h), we get

$$\Gamma_{|\phi|^4} = -u^2 \frac{5\pi}{c^3} K_d \Lambda^{d-3} l + ub^2 \frac{54\pi}{c^5} K_d \Lambda^{d-5} l + g^4 \frac{N\pi}{2v^3} K_d \Lambda^{d-3} l - b^4 \frac{405\pi}{4c^7} K_d \Lambda^{d-7} l. \quad (10)$$

Combining all vertices, we arrive at a set of RG equations for coupling constants,

$$\frac{d\tilde{g}^2}{dl} = (3-d)\tilde{g}^2 - \frac{9\pi\tilde{b}^2\tilde{g}^2}{4c^5} - \left[\frac{N\pi}{2v^3} + \frac{2\pi}{c(c+v)^2} \right] \tilde{g}^4, \quad (11)$$

$$\frac{d\tilde{b}^2}{dl} = (5-d)\tilde{b}^2 - \frac{3N\pi}{2v^3} \tilde{g}^2 \tilde{b}^2 - \frac{6\pi}{c^3} \tilde{b}^2 \tilde{u} - \frac{27\pi}{4c^5} \tilde{b}^4, \quad (12)$$

$$\frac{d\tilde{u}}{dl} = (3-d)\tilde{u} - \frac{N\pi\tilde{g}^2\tilde{u}}{v^3} + \frac{N\pi\tilde{g}^4}{v^3} - \frac{5\pi\tilde{u}^2}{c^3} + \frac{99\pi\tilde{u}\tilde{b}^2}{2c^5} - \frac{405\pi\tilde{b}^4}{4c^7}. \quad (13)$$

B. Prediction of FIQCP for $N > 1/2$ and $d = 2$

When $d = 2$ (two spatial dimensions), the RG flow equations above have a fixed point $(\tilde{g}^{*2}, \tilde{u}^*, \tilde{b}^{*2}) = (\frac{2}{(N+1)\pi}, \frac{\sqrt{N^2+38N+1}-N+1}{10(N+1)\pi}, 0)$ where we have set $v^* = c^* = 1$ for simplicity, since the boson velocity and the fermion velocity flow to the same value at this fixed point as discussed above. In the large- N limit, the fixed point approaches $(\frac{2}{N\pi}, \frac{2}{N\pi}, 0)$, well controlled by $\frac{1}{N}$. A crucial feature is the emergent $O(2)$ symmetry at this fixed point, in other words, the cubic terms of order parameter vanish, which is essential for the occurrence of a continuous transition point. By linearizing RG equations near this fixed point, we get the eigenvalues of the scaling matrix, which are $(-1, -\frac{\sqrt{N^2+38N+1}}{N+1}, \frac{3(N+4-\sqrt{N^2+38N+1})}{5(N+1)})$. The first two are always negative while the last eigenvalue is positive for $N > \frac{1}{2}$ and negative for $N < \frac{1}{2}$, indicating

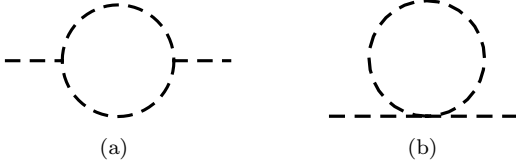


FIG. 4. One-loop Feynman diagrams that renormalize the boson mass.

the existence of a critical N_c such that if the number of Dirac fermion flavors N are larger than N_c , the fixed point is stable. The RG flow diagrams in the coupling constants space spanned by $(\tilde{g}^{*2}, \tilde{u}^*, \tilde{b}^{*2})$ as shown in Fig. 3 for $N=3$, indicates a stable fixed point consistent with our analysis.

A stable fixed point at critical surface $r = r_c(\tilde{g}^2, \tilde{u}, \tilde{b}^2)$ corresponds to a genuine critical point. Since we have identified a stable fixed point with zero \tilde{b}^{*2} , i.e., no cubic terms present in this fixed point, it clearly corresponds to a FIQCP. Presence of gapless Dirac fermions at the transition point renders the cubic term irrelevant and results in a FIQCP beyond Landau criterion. The critical exponent η at this critical point is given by $\eta = 2\eta_\phi = \frac{N}{N+1}$. Due to the existence of Dirac fermions at the transition, the dynamical exponent is $z=1$ and fermion anomalous dimension is $\eta_\psi = \frac{1}{4(N+1)}$. The non-vanishing fermion anomalous dimension indicates breakdown of quasiparticle picture and the critical theory is strongly interacting.

To explore other critical behavior, we also calculate the renormalization of boson mass (the relevant direction). Relevant Feynman diagrams are shown in Fig. 4. A straight forward calculation gives the boson self-energy that renormalizes mass term, i.e.,

$$\Gamma_{|\phi|^2} = \left(-\frac{9\pi\tilde{b}^2}{(1+\tilde{r})^{3/2}} + \frac{4\pi\tilde{u}}{\sqrt{1+\tilde{r}}} \right) \Lambda^2 l, \quad (14)$$

where $\tilde{r} \equiv \Lambda^{-2}r$ is a dimensionless constant. The scaling dimension for r at the critical point is $y_r = 2 - 2\eta_\phi + \frac{27\pi\tilde{b}^{*2}}{2} - 2\pi\tilde{u}^* = 2 - \frac{\sqrt{N^2+38N+1}+4N+1}{5(N+1)}$ and the critical exponent is given by $\nu^{-1} = y_r$. Other critical exponents can be determined by hyperscaling relations.

C. No FIQCP for any N and $d = 3$

Having explored the properties of FIQCP in 2+1D, we discuss the RG analysis in generic dimensions. A mean field theory yields the upper critical dimension for the existence of such a FIQCP is $d_{\text{MF}} \equiv 2$ (Ref. [7]). Our RG result is consistent with the mean field calculation since the mean field result corresponds to $N \rightarrow \infty$. Inclusion of quantum fluctuations for finite N beyond mean field theory will lead to a correction to the upper critical dimension as we discuss below.

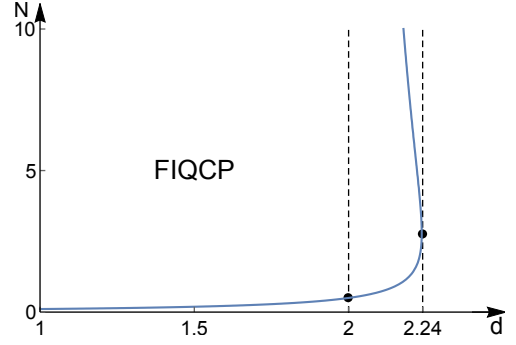


FIG. 5. Critical number of Dirac fermion flavors at d dimensions. The curve corresponds to $N_l(d)$ and $N_u(d)$ and separates the region where FIQCP occurs. $N_l(2) = \frac{1}{2}$ and $d_c \approx 2.24$ are shown explicitly on the curve.

The solution of the RG equations in $d < 3$ dimensions with both $\tilde{g}^{*2} > 0$ and $\tilde{u}^* > 0$ reads

$$\tilde{g}^{*2} = \frac{2(3-d)}{(N+1)\pi}, \quad (15)$$

$$\tilde{b}^{*2} = 0, \quad (16)$$

$$\tilde{u}^* = \frac{(3-d)(\sqrt{N^2+38N+1}-N+1)}{10(N+1)\pi}. \quad (17)$$

To determine whether it corresponds to a FIQCP, we calculate the eigenvalues of scaling matrix by linearizing the RG equations near the fixed point. They are given by $(d-3, \frac{(d-3)\sqrt{N^2+38N+1}}{N+1}, \frac{16-11N+d(7N-2)-3(d-3)\sqrt{N^2+38N+1}}{5(N+1)})$. Although the first two scaling fields are always negative when $d < 3$, the third scaling field could be positive depending on the number of flavors N and the dimensionality d . We find a lower bound for the number of Dirac fermions in d dimensions, i.e.,

$$N_l(d) = \frac{37d^2 - 232d + 343 - 9\sqrt{(d-3)(17d^2 - 110d + 161)}}{8d^2 - 20d + 8}, \quad (18)$$

and also an upper bound when $d > d_{\text{MF}}$,

$$N_u(d) = \frac{37d^2 - 232d + 343 + 9\sqrt{(d-3)(17d^2 - 110d + 161)}}{8d^2 - 20d + 8}. \quad (19)$$

The existence of an upper bound in $d > d_{\text{MF}}$ is expected from the mean field results since the RG calculation should reproduce mean field result when $N \rightarrow \infty$. These two bounds merge at $d_c = \frac{55-12\sqrt{2}}{17} \approx 2.24$. The region for FIQCP to happen is shown in Fig. 5. Thus, for $d = 3$, Dirac fermions cannot induce a FIQCP for any N . Nonetheless, it was shown that a FIQCP can occur at Z_3 nodal-nematic transitions in 3+1D double-Weyl systems [43].

IV. AN EXACT RESULT: NO FIQCP FOR $N = 1/2$ AND $d = 2$

We now prove an exact result of the absence of FIQCP for $N = 1/2$ and $d = 2$ by employing the results of space-time supersymmetry. The low energy physics of Majorana fermions on a honeycomb lattice with next-neighbor hopping is equivalent to a single two-component Dirac fermion or half flavor of four-component Dirac fermions ($N = \frac{1}{2}$). The non-interacting hamiltonian reads

$$H = \sum_{\langle xx' \rangle} (it_{xx'} \gamma_x \gamma_{x'} + h.c.), \quad (20)$$

where $\langle xx' \rangle$ represents hopping between nearest-neighbor (NN) sites, the NN hopping amplitude $t_{xx'} = t$, γ_x is Majorana operator at site x satisfying $\gamma^\dagger = \gamma$ and $\{\gamma_x, \gamma_{x'}\} = 2\delta_{xx'}$. Using Fourier transformation, $\gamma_x = \frac{1}{\sqrt{2M}} \sum_{\vec{k}} \gamma_{\vec{k}} e^{i\vec{k} \cdot \vec{x}}$, where M is the site number, one obtains

$$H = \frac{t}{2} \sum_{\vec{k}} [i\gamma_{A,-\vec{k}} \gamma_{B,\vec{k}} (1 + e^{-i\vec{k} \cdot \vec{e}_1} + e^{-i\vec{k} \cdot \vec{e}_2}) + h.c.], \quad (21)$$

where $\vec{e}_1 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$, $\vec{e}_2 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$ are lattice constants, and A, B are sublattice indices. Diagonalizing the Hamiltonian, one gets two Majorana cones ψ_{\pm} locating at $\pm \vec{K} = (\pm \frac{4\pi}{3}, 0)$ point. The low-energy non-interacting Hamiltonian is given by

$$\mathcal{H} = \psi^\dagger v (i\sigma^x \partial_y - i\sigma^y \tau^z \partial_x) \psi, \quad (22)$$

where $\psi \equiv (\psi_+, \psi_-)^T$, $v \equiv \frac{\sqrt{3}t}{4}$ is Fermi velocity and σ is Pauli matrix with sublattice indices A/B .

Note that only half of Majorana fermions in k -space are independent, since $\gamma_{\vec{k}}^\dagger = \gamma_{-\vec{k}}$. In other words, two Majorana cones are related through $\psi_- = \psi_+^\dagger$ such that the two Majorana cones can be combined into a single Dirac cone. Then, we consider the Kekule-VBS order in hopping $t_{xx'}$, for which the order parameter is given by $\phi \propto \psi_-^\dagger \sigma^y \psi_+ \equiv \psi_+^T \sigma^y \psi_+$, which is equivalent to intra-valley ‘‘pairing’’ of a single Dirac cone. Note that there is not $U(1)$ symmetry in such theory because of the real nature of Majorana fermions. Instead of being a pairing order parameter, ϕ actually breaks the translational symmetry of the honeycomb lattice since it carries a finite momentum $2\vec{K}$ and it is a Z_3 symmetry breaking.

We first assume that the phase transition is continuous by setting $b=0$ to get a fixed point theory, and then determine the fate of cubic terms at this fixed point. Setting $b=0$, the effective theory at the assumed critical point is

$$\mathcal{L} = \psi^\dagger [\partial_\tau + v(i\sigma^x \partial_y - i\sigma^y \tau^z \partial_x)] \psi + |\partial_\tau \phi|^2 + c^2 \sum_i |\partial_i \phi|^2 + \frac{g}{2} (\phi \psi^T \sigma^y \psi + h.c.) + u |\phi|^4, \quad (23)$$

where ψ represents a single two-component Dirac fermion. The factor $\frac{1}{2}$ in the coupling constant g comes

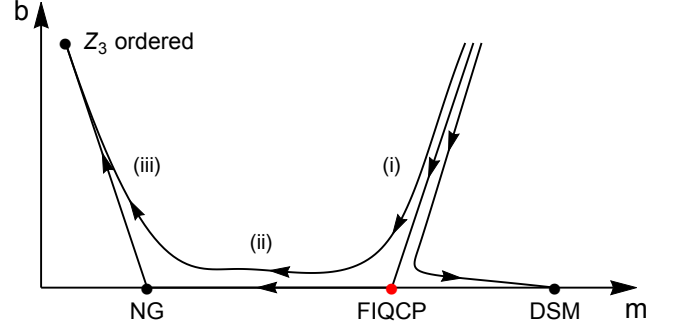


FIG. 6. A schematic global RG flow diagram. Here the x -axis (y -axis) represents the tuning parameter m (Z_3 anisotropy b). The arrows indicate RG flow directions. SM and VBS denote Dirac semimetal and Z_3 ordered phase, respectively. FIQCP is the transition point between semimetal and Z_3 ordered phase. NG means Nambu-Goldstone fixed point. Region (i), (ii), and (iii) denote those regions where scaling behavior is controlled in perturbative RG calculations, e.g., $b^2 \ll 1$.

from that the fact that we rescale ψ to $\frac{\psi}{\sqrt{2}}$ when combine two Majorana cones into one Dirac cone. Setting $N = \frac{1}{2}$ in the RG flow equations, one obtains the following fixed point: $c^* = v^*$, $(\frac{g^*}{2})^2 = u^*$. Such critical theory has emergent SUSY and corresponds to the $\mathcal{N} = 2$ Wess-Zumino model [44, 45, 63, 64].

We have shown earlier that when $N = \frac{1}{2}$ the cubic terms are marginal at the one-loop level, and could be marginally relevant or irrelevant if higher-order quantum fluctuations are included. Interestingly, owing to the $U(1)_{\mathcal{R}}$ symmetry in the $\mathcal{N} = 2$ Wess-Zumino model, the scaling dimension of chiral multiplets is known *exactly* [46, 47], and is equal to the \mathcal{R} -charge. Thus, the scaling dimension of ϕ^3 at the assumed fixed point is known exactly: $\Delta_{\phi^3} = 2$, which is much smaller than 3. As a result, the cubic term is strongly relevant and the assumed fixed point is unstable due to the relevance of cubic perturbations. Namely, $N = \frac{1}{2}$ flavor of 4-component Dirac fermions cannot induce a FIQCP at the Z_3 Gross-Neveu transition. In other words, the critical number of flavors of Dirac fermions satisfies $N_c > \frac{1}{2}$. This inequality is *exact*!

V. TWO LENGTH SCALES AT FIQCP

The existence of dangerously irrelevant fields at a quantum critical point can yield two divergent length scales [51–57] and different critical exponents in the two sides of the transition point. A schematic phase diagram is shown in Fig. 6, where the x -axis and y -axis represent tuning parameter m and Z_3 anisotropy b , respectively, and the arrows indicate RG flow directions, i.e., flow to low energy and long wavelength. There are two stable fixed points corresponding to Dirac semimetal phase and Z_3 ordered phase while FIQCP is the transition point

between them. Moreover, there is another unstable fixed point, the Nambu-Goldstone (NG) fixed point, locating at $b=0$ axis. If the system is $O(2)$ symmetric, the NG fixed point describes the gapless phase with NG modes resulted from spontaneous breaking of continuous $O(2)$ symmetry in the ordered phase. An effective Lagrangian for the NG phase reads

$$\mathcal{L} = \frac{1}{2} \sum_{\mu=\tau,x,y} (\partial_\mu \theta)^2, \quad (24)$$

where $\phi \equiv |\phi|e^{i\theta}$. However, since the cubic term b explicitly breaks the $O(2)$ symmetry, b is then relevant at the NG fixed point and drives the system into Z_3 ordered phase where the NG modes are gapped.

When the system moves into the ordered phase and locates slightly away from critical point with $m \lesssim m_c$, along the RG flow the system enters region (i), where the scaling behavior in the crossover from region (i) to (ii) is controlled by the fixed point FIQCP. Here, the characteristic length scale is given by the correlation length of the Z_3 order, ξ , with $\xi \sim (m_c - m)^{-\nu}$. As the Z_3 anisotropy is dangerously irrelevant at the FIQCP, it is expected that $b^2 \ll 1$ and the system is dominated by the pseudo-NG modes with small masses in region (ii). Flowing to lower energy and longer distance, the system enters region (iii), where the scaling behavior in the crossover from region (ii) to (iii) is controlled by the NG fixed point instead. Now that the Z_3 anisotropy is relevant at NG fixed point, the pseudo-NG modes are damped and different domains of Z_3 ordering become apparent. Then, another characteristic length scale emerges and is given by the diameter of domains, ξ' , with $\xi' \sim (m_c - m)^{-\nu'}$. These two length scales obey different scaling laws [53],

$$\frac{\nu'}{\nu} = 1 - \frac{y_{b^2}}{2}, \quad (25)$$

$$= 1 + \frac{3(\sqrt{N^2 + 38N + 1} - N - 4)}{10(N + 1)}, \quad (26)$$

where ν (ν') is critical exponent capturing the divergence of characteristic length scales ξ (ξ') approaching towards criticality from the ordered side, and y_{b^2} is the scaling dimension of b^2 at the FIQCP. When the FIQCP happens (namely $N > N_c > 1/2$), one can see that $\nu' > \nu$. The domain wall length scale is larger than the correlation length, consistent with our analysis of RG flows above. Note that ν' only exists in ordered phase while the same ν can be measured on both sides of transition.

We would like to mention that the emergence of two divergent length scales around the FIQCP differs from the one occurring in the usual bosonic Z_n theory due to the existence of gapless electrons at the FIQCP. In conventional bosonic Z_n theory, the Z_n anisotropy is irrelevant only for $n \geq 4$ from $4-\epsilon$ RG calculations [51, 53, 73]. Here, owing to the presence of gapless Dirac fermions at transition, the Z_3 anisotropy is rendered dangerously irrelevant at FIQCP. Moreover, the ratio of $\frac{\nu'}{\nu}$ for the FIQCP depends on the number of flavors of Dirac fermions, as expected.

VI. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have presented a RG study of the Z_3 Gross-Neveu quantum phase transitions in Dirac systems whose low-energy effective field theory contains cubic terms of order-parameters. Our RG results show that for $d = 2$ there exists finite range of N , namely $N > N_c$, such that the putative first-order phase transitions can be driven to a continuous phase transition due to the fluctuations of gapless Dirac fermions at the transition point. Furthermore, with the help of spacetime SUSY in 2+1D, we obtain an exact lower bound for the critical number of flavors of Dirac fermions to realize a FIQCP, namely $N_c > \frac{1}{2}$. Moreover, the large-scale sign-problem-free QMC simulations [7, 74] indicate that $N_c < 2$. We conclude that $\frac{1}{2} < N_c < 2$. The existence of a FIQCP provides an exotic scenario to realize type-II Landau-forbidden transitions.

We observe that there is a common feature shared by Landau-forbidden phase transitions (e.g. FIQCP and DQCP): the existence of dangerously irrelevant fields. In the language of RG, the Landau criterions can be interpreted by the existence of more than one relevant parameters rendering a first-order phase transition or a multicritical point, rather than a generic quantum critical point. For instance, the ϕ^3 terms in the case of FIQCP and the quartic monopole creation operator in the case of DQCP are seemingly relevant operators (besides the usual relevant mass term of the order parameter fields). However, these seemingly relevant operators are rendered irrelevant at the critical points by either gapless fermions in the case of FIQCP or gapless deconfined spinons in the case of DQCP; as a result, the critical theory exhibits an emergent $O(2)$ symmetry. Presence of dangerously irrelevant operators around the transition point can reduce the emergent symmetry, leading to the emergence of two length scales. We also explored the values of different critical exponents by one-loop RG calculations.

Possible experiments realizing a FIQCP include the quantum phase transitions between the semi-metallic graphene [75, 76] and the Kekule-VBS phase [77] by growing the graphene on certain substrate or by tuning the strain. There are also experiments on adsorbed monolayer quantum gases, including Helium, on the top of graphene-like substrates [78, 79], where the quantum gas films can undergo a phase transition to the famous $\sqrt{3} \times \sqrt{3}$ order, which is a Z_3 Gross-Neveu transition.

Acknowledgments: We would like to thank Yi-Fan Jiang, Zi-Xiang Li, and Shi-Xin Zhang for discussions. This work was supported in part by the National Thousand-Young-Talents Program (HY) and the NSFC under Grant No. 11474175 (SKJ and HY).

Appendix A: Trace properties for the Gamma matrices

As defined in the main text, γ^μ , $\mu = 1, \dots, d+2$, are the gamma matrices satisfying $\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}$ and $\gamma^\pm = \frac{1}{2}(\gamma^{d+1} \pm i\gamma^{d+2})$. In the following, we use Greek letter to denote 1, ..., d and English letter to denote \pm . The Gamma matrices defined in the main text have the properties, $\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}$, $\{\gamma^i, \gamma^j\} = 2g^{ij}$, $\{\gamma^i, \gamma^\mu\} = 0$, where $g^{ij} = \frac{1}{2}(1 - \delta^{ij})$. The trace of Gamma matrices are given by

$$\begin{aligned}\text{Tr}[\gamma^i \gamma^\mu \gamma^j \gamma^\nu] &= -4N g^{ij} \delta^{\mu\nu} \\ \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] &= 4N [\delta^{\mu\nu} \delta^{\rho\sigma} - \delta^{\mu\rho} \delta^{\nu\sigma} + \delta^{\mu\sigma} \delta^{\nu\rho}] \\ \text{Tr}[\gamma^i \gamma^\mu \gamma^j \gamma^\nu \gamma^k \gamma^\rho] &= 0 \\ \text{Tr}[\gamma^i \gamma^\mu \gamma^j \gamma^\nu \gamma^k \gamma^\rho \gamma^l \gamma^\sigma] &= 4N [\delta^{\mu\nu} \delta^{\rho\sigma} - \delta^{\mu\rho} \delta^{\nu\sigma} + \delta^{\mu\sigma} \delta^{\nu\rho}] \\ &\quad \times [g^{ij} g^{kl} - g^{ik} g^{jl} + g^{il} g^{jk}]\end{aligned}$$

where we only consider 4×4 Gamma matrices since we are interested in $d = 2, 3$, and promote the flavor of Dirac fermion to be N .

Appendix B: Renormalization of boson mass term in Dirac semimetals

The Feynman diagrams contributing to boson mass renormalization are shown in Fig. 4 in the main text.

A straightforward evaluation leads to

$$\begin{aligned}\Gamma_{|\phi|^2} &= -18b^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + r)^2} + 4u \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + r}, \\ &= \left(-\frac{9\pi\tilde{b}^2}{(1+\tilde{r})^{3/2}} + \frac{4\pi\tilde{u}}{\sqrt{1+\tilde{r}}} \right) \Lambda^2 l,\end{aligned}\quad (\text{B1})$$

where the various dimensionless coupling constants are defined in the main text. Having this two vertex, the renormalization group (RG) equation for boson mass is given by

$$\begin{aligned}\frac{d\tilde{r}}{dl} &= 2\tilde{r} - \frac{N\pi\tilde{g}^2\tilde{r}}{2} - \frac{9\pi\tilde{b}^2\tilde{r}}{4(1+\tilde{r})^{5/2}} \\ &\quad - \frac{9\pi\tilde{b}^2}{(1+\tilde{r})^{3/2}} + \frac{4\pi\tilde{u}}{\sqrt{1+\tilde{r}}}.\end{aligned}\quad (\text{B2})$$

As a first approximation, the solution at the fermion-induced quantum critical point (FIQCP) is obtained by substituting the fixed point coupling constants to above RG equation and the result yields $r \approx -\frac{8}{N}$.

-
- [1] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, (Oxford, Pergamon, 1958).
 - [2] E. M. Lifshitz, *J. Physique*, **6**, 61 (1942).
 - [3] K. Binder, *Rep. Prog. Phys.* **50**, 783 (1987).
 - [4] H. W. Blöte and R. H. Swendsen, *Phys. Rev. Lett.* **43**, 799 (1979).
 - [5] R. J. Baxter, *J. Phys. C, Solid State Phys*, **6**, L445 (1973).
 - [6] F. Y. Wu, *Rev. Mod. Phys.* **54**, 235 (1982).
 - [7] Z.-X. Li, Y.-F. Jiang, S.-K. Jian, and H. Yao, *arXiv:1512.07908* (2015).
 - [8] Z.-X. Li, Y.-F. Jiang, and H. Yao, *Phys. Rev. B* **91**, 241117 (2015).
 - [9] Z.-X. Li, Y.-F. Jiang, and H. Yao, *arXiv:1601.05780* (2016).
 - [10] D. J. Gross and A. Neveu, *Phys. Rev. D* **10**, 3235 (1974).
 - [11] S. Coleman, *Aspects of symmetry: selected Erice lectures*, Cambridge University Press (1988).
 - [12] B. Rosenstein, H.-L. Yu, and A. Kovner, *Phys. Lett. B* **314**, 381 (1993).
 - [13] M. Moshe and J. Zinn-Justin, *Physics Reports*, **385**, 69 (2003).
 - [14] I. F. Herbut, *Phys. Rev. Lett.* **97**, 146401 (2006).
 - [15] M. M. Scherer and I. F. Herbut, *arXiv:1609.03208* (2016).
 - [16] S. Sachdev, *Quantum Phase Transitions*, Cambridge University Press, Cambridge (2011).
 - [17] P. W. Anderson, *Science* **235**, 1196 (1987).
 - [18] I. Affleck and J. B. Marston, *Phys. Rev. B* **37**, 3774 (1988).
 - [19] W. Rantner and X.-G. Wen, *Phys. Rev. Lett.* **86**, 3871 (2001).
 - [20] X.-G. Wen, *Phys. Rev. B* **65**, 165113 (2002).
 - [21] X.-G. Wen, *Quantum field theory of many-body systems*, Oxford University Press (2004).
 - [22] S. Ryu, O. I. Motrunich, J. Alicea, and M. P. A. Fisher, *Phys. Rev. B* **75**, 184406 (2007).
 - [23] Y. Ran, M. Hermele, P. A. Lee, and X.-G. Wen, *Phys. Rev. Lett.* **98**, 117205 (2007).
 - [24] H. Yao, S.-C. Zhang, and S. A. Kivelson, *Phys. Rev. Lett.* **102**, 217202 (2009).
 - [25] H. J. Liao, Z. Y. Xie, J. Chen, Z. Y. Liu, H. D. Xie, R. Z. Huang, B. Normand, and T. Xiang, *arXiv:1610.04727* (2016).
 - [26] P. Corboz, M. Lajko, A. M. Lauchli, K. Penc, and F. Mila, *Phys. Rev. X* **2**, 041013 (2012).
 - [27] A. M. Polyakov, *Nucl. Phys. B* **120**, 429 (1977).
 - [28] M. Hermele, T. Senthil, M. P. A. Fisher, P. A. Lee, N. Nagaosa, and X.-G. Wen, *Phys. Rev. B* **70**, 214437 (2004).
 - [29] F. F. Assaad, *Phys. Rev. B* **71**, 075103 (2004).
 - [30] F. S. Nogueira and H. Kleinert, *Phys. Rev. Lett* **95**, 176406 (2005).
 - [31] C. Xu, *Phys. Rev. B* **78**, 054432 (2008).
 - [32] T. Grover, *Phys. Rev. Lett.* **112**, 151601 (2014).
 - [33] R. D. Pisarski, *Phys. Rev. D* **29**, 2423 (1984).

- [34] T. W. Appelquist, M. Bowick, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. D **33**, 3704 (1986).
- [35] T. Appelquist, D. Nash, and L. C. R. Wijewardhana, Phys. Rev. Lett. **60**, 2575 (1988).
- [36] D. Nash, Phys. Rev. Lett. **62**, 3024 (1989).
- [37] K.-I. Kondo, T. Ebihara, T. Iizuka, and E. Tanaka, Nucl. Phys. B **434**, 85 (1995).
- [38] I. J. R. Aitchison, N. E. Mavromatos, and D. O. McNeill, Phys. Lett. B **402**, 154 (1997).
- [39] I. F. Herbut, Phys. Rev. B **66**, 094504 (2002).
- [40] C. S. Fischer, R. Alkofer, T. Dahm, and P. Maris, Phys. Rev. D **70**, 073007 (2005).
- [41] A. Bashir, A. Raya, I. C. Cloet, and C. D. Roberts, Phys. Rev. C **78**, 055201 (2008).
- [42] C. Strouthos and J. B. Kogut, *Journal of Physics: Conference Series* **150**, 052247, IOP Publishing (2009).
- [43] S.-K. Jian and H. Yao, arXiv:1609.06313 (2016).
- [44] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press (1992).
- [45] S. Weinberg, *The Quantum Theory of Fields: Supersymmetry*, Cambridge University Press, Cambridge (2000).
- [46] N. Seiberg, PASCOS 357 (1994).
- [47] O. Aharony, A. Hanany, K. Intriligator, N. Seiberg, and M. J. Strassler, Nucl. Phys. B **499**, 67 (1997).
- [48] M. J. Strassler, arXiv:hep-th/0309149 (2003).
- [49] D. R. Nelson, Phys. Rev. B **13**, 2222 (1976).
- [50] S. Miyashita, J. Phys. Soc. Jpn. **66**, 3411 (1997).
- [51] M. Oshikawa, Phys. Rev. B **61**, 3430 (2000).
- [52] J. M. Carmona, A. Pelissetto, and E. Vicari, Phys. Rev. B **61**, 15136 (2000).
- [53] T. Okubo, K. Oshikawa, H. Watanabe, and N. Kawashima, Phys. Rev. B **91**, 174417 (2015).
- [54] F. Leonard, and B. Delamotte, Phys. Rev. Lett. **115**, 200601 (2015).
- [55] H. Shao, W. Guo, and A. W. Sandvik, Science **352**, 213 (2016).
- [56] T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science **303**, 1490 (2004).
- [57] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Phys. Rev. B **70**, 144407 (2004).
- [58] J. Lou, A. W. Sandvik, and L. Balents, Phys. Rev. Lett. **99**, 207203 (2007).
- [59] P. Ghaemi, S. Ryu, and D. H. Lee, Phys. Rev. B **81**, 081403(R) (2010).
- [60] R. K. Kaul, R. G. Melko, and A. W. Sandvik, Annual Review of Condensed Matter Physics **4**, 179 (2013).
- [61] A. Nahum, J. T. Chalker, P. Serna, M. Ortun, and A. M. Somoza, Phys. Rev. X **5**, 041048 (2015).
- [62] B. Roy, V. Juricic, and I. F. Herbut, Phys. Rev. B **87**, 041401(R) (2013).
- [63] T. Grover, D. N. Sheng, and A. Vishwanath, Science **344**, 280 (2014).
- [64] P. Ponte and S.-S. Lee, New J. Phys. **16**, 013044 (2014).
- [65] S.-K. Jian, Y.-F. Jiang, and H. Yao, Phys. Rev. Lett. **114**, 237001 (2015).
- [66] S.-K. Jian, C.-H. Lin, J. Maciejko, and H. Yao, arXiv:1609.02146 (2016).
- [67] N. Zerf, C.-H. Lin, and J. Maciejko, arXiv:1605.09423 (2016).
- [68] T. H. Hsieh, G. B. Halasz, and T. Grover, Phys. Rev. Lett. **117**, 166802 (2016).
- [69] O. Vafek, Z. Teseanovic, and M. Franz, Phys. Rev. Lett. **89**, 157003 (2002).
- [70] H. Isobe and N. Nagaosa, Phys. Rev. B **68**, 165127 (2012).
- [71] B. Roy, V. Juricic, and I. F. Herbut, JHEP **04**, 018 (2016).
- [72] S. Ryu, C. Mudry, C.-Y. Hou, and C. Chamon, Phys. Rev. B **80**, 205319 (2009).
- [73] M. Hasenbusch and E. Vicari, Phys. Rev. B **84**, 125136 (2011).
- [74] Z. Zhou, D. Wang, Z. Y. Meng, Y. Wang, and C. Wu, Phys. Rev. B **93**, 245157 (2016).
- [75] K. S. A. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, Nature **438**, 197 (2005).
- [76] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. **81**, 109 (2009).
- [77] C. Gutierrez, C.-J. Kim, L. Brown, T. Schiros, D. Nordlund, E. B. Lochocki, K. M. Shen, J. Park, and A. N. Pasupathy, Nature Physics **12**, 950 (2016).
- [78] M. Bretz, Phys. Rev. Lett. **38**, 501 (1976).
- [79] L. Reatto, D. E. Galli, M. Nava, and M. W. Cole, *Journal of Physics: Condensed Matter* **25**, 443001 (2013).